PANIC05, Santa Fe
October 24, 2005

f_D and f_{D_s} in Lattice QCD with Exact Chiral Symmetry

Ting-Wai Chiu

Physics Dept., National Taiwan Univ.

Outline:

1. Introduction
2. Quark Propagators with Exact Chiral Symmetry
3. Pseudoscalar Masses and Decay Constants
4. Summary and Concluding Remarks

Ref: TWC, T.H. Hsieh, J.Y. Lee, P.H. Liu, H.J. Chang, Phys.Lett.B624(2005)31; Proc.Sci.LAT2005(2005)041.

Introduction

The leptonic decay constants (e.g., f_D, f_{D_s}, f_B and f_{B_s}) play an important role in extracting the CKM matrix elements which are crucial for testing the flavor sector of the standard model via the unitarity of CKM matrix.

Precise determinations of f_{D^+} and $f_{D_s^+}$ will result from the high-statistics program of CLEO-c, however, the determination of f_B and f_{B_s} remains beyond the reach of current experiments. Thus lattice QCD determinations of f_B and f_{B_s} are of fundamental importance.

Obviously, the first step for lattice QCD is to check whether lattice determinations of f_{D^+} and $f_{D_s^+}$ can give a reliable prediction of the values coming from CLEO-c.

Introduction (cont)

One of the basic objectives of lattice QCD is to compute hadron masses (and decay constants) nonperturbatively from the first principles.

For hadrons only composed of s and c quarks, their masses and decay constants can be measured directly with accessible lattice sizes.

However, for hadrons containing u, d light quarks, the performance of the present generation of computers is still inadequate for computing their masses at the physical limit ($m_\pi \simeq 135$ MeV). Thus chiral extrapolation is required.

Introduction (cont)

A strategy for hadrons containing u,d,s,c:

- Compute time-correlation functions
- From $C_\pi(t)$, $f_\pi = 131$ MeV fixes a^{-1}
- From $C_\rho(t)$, $\phi(1020)$ fixes m_s, $J/\psi(3097)$ fixes m_c
- For hadrons containing u, d quarks, chiral extrap. to $m_\pi = 135$ MeV

Then the masses and other phys. quantities of any hadrons containing c, s, u, d quarks are predictions of QCD from the first principles.

Introduction (cont)

Theoretically, the proper way to proceed is to use lattice fermions which preserve chiral sym exactly at finite a. Then the quark propagator is in the form $(D_c + m_q)^{-1}$, where D_c satisfies $D_c\gamma_5 + \gamma_5 D_c = 0$, and

$$(D_c + m_q)^{-1} \xrightarrow{a \to 0} [\gamma_\mu(\partial_\mu + iA_\mu) + m_q]^{-1}$$

Thus the bare quark mass m_q is well-defined for any gauge configurations.

Optimal Domain-Wall Fermion [TWC,'02,'03]

$$\mathcal{A}_f = \sum_{s,s'=0}^{N_s+1} \sum_{x,x'} \bar{\psi}(x,s)[(1+\omega_s D_w)_{x,x'}\delta_{s,s'} \\ -(1-\omega_s D_w)_{x,x'}(P_-\delta_{s',s+1} + P_+\delta_{s',s-1})]\psi(x',s')$$

with boundary conditions (m_q : bare quark mass)

$$\begin{aligned} P_+\psi(x,-1) &= -r m_q P_+\psi(x,N_s+1), \quad r = \frac{1}{2m_0} \\ P_-\psi(x,N_s+2) &= -r m_q P_-\psi(x,0), \quad P_\pm = \frac{1 \pm \gamma_5}{2} \end{aligned}$$

where D_w is the Wilson Dirac op. plus $-m_0 \in (-2,0)$, and $\{\omega_s\}$ are weights specified by the exact formula

$$\begin{aligned} \omega_s &= \frac{1}{\lambda_{min}}\sqrt{1-\kappa'^2 \mathrm{sn}^2(v_s;\kappa')}, \quad s = 1,\cdots,N_s \\ \omega_0 &= \omega_{N_s+1} = 0 \end{aligned}$$

such that D_c possesses the maximal chiral sym for any given N_s and gauge background

Optimal Domain-Wall Fermion (cont)

Quark fields are defined by the boundary modes:

$$\begin{aligned} q(x) &= \sqrt{r}[P_-\psi(x,0) + P_+\psi(x,N_s+1)] \\ \bar{q}(x) &= \sqrt{r}[\bar{\psi}(x,0)P_+ + \bar{\psi}(x,N_s+1)P_-] \end{aligned}$$

After introducing pseudofermions with $m_q = 2m_0$, the generating function for n-point function of q and $\bar{q}$ is

$$Z[J,\bar{J}] = \frac{\int [dU] e^{-\mathcal{A}_G[U]} \det D(m_q)\ e^{\bar{J}(D_c+m_q)^{-1}J}}{\int [dU] e^{-\mathcal{A}_G[U]} \det D(m_q)}$$

where $\mathcal{A}_g$ is the gauge action, and

$$\begin{aligned} D(m_q) &= (D_c + m_q)(1 + rD_c)^{-1} \\ D_c &= 2m_0 \frac{1+\gamma_5 S(H_w)}{1-\gamma_5 S(H_w)} \\ S(H_w) &= \frac{1 - \prod_{s=0}^{N_s+1} T_s}{1 + \prod_{s=0}^{N_s+1} T_s} \xrightarrow{N_s\to\infty} \frac{H_w}{\sqrt{H_w^2}} \Rightarrow D_c\gamma_5 + \gamma_5 D_c = 0 \\ T_s &= \frac{1-\omega_s H_w}{1+\omega_s H_w}, \quad H_w = \gamma_5 D_w, \end{aligned}$$

The quark propagator in background gauge field is

$$\langle q(x)\bar{q}(y)\rangle = - \left. \frac{\delta^2 Z[J,\bar{J}]}{\delta \bar{J}(x)\delta J(y)} \right|_{J=\bar{J}=0} = (D_c+m_q)^{-1}_{x,y}$$

A Scheme to Compute Propagators

Since

$$(D_c + m_q)^{-1} = \left(1 - \frac{m_q}{2m_0}\right)^{-1} \left[D^{-1}(m_q) - \frac{1}{2m_0}\right]$$

where

$$D(m_q) = m_q + (m_0 - m_q/2)[1 + \gamma_5 S(H_w)]$$

Thus $(D_c + m_q)^{-1}$ can be obtained by solving

$$D(m_q)Y = \mathbb{1}$$

with nested conjugate gradient, which turns out to be highly efficient (in terms of precision of chirality vs. CPU time and memory storage) if the inner CG loop is iterated with Neuberger's double pass algorithm.

[Neuberger,'98, TWC & Hsieh,'03]

Lattice setup

We generate 100 gauge confs with single plaquette action at $\beta = 6.1$ on $20^3 \times 40$ lattice.

Fixing $m_0 = 1.3$, we project out 16 low-lying eigenmodes of $|H_w|$ and perform the nested conjugate gradient in the complement of the vector space spanned by these eigenmodes.

For $N_s = 128$, the weights $\{\omega_s\}$ are fixed with

$\lambda_{min} = 0.18$ and $\lambda_{max} = 6.3$ for all gauge confs.

Quark Propagators with Exact Chiral Sym

For each conf, quark prop. are computed for 30 quark masses in the range $0.03 \le m_q a \le 0.8$, with stopping criteria 10^{-11} (2×10^{-12}) for outer (inner) CG.

Then chiral sym breaking due to finite $N_s (= 128)$ is

$$\sigma = \left| \frac{Y^\dagger S^2 Y}{Y^\dagger Y} - 1 \right| < 10^{-14}$$

The norm of the residual vector for each column is

$$||(D_c + m_q)Y - 1\!\!\mathrm{I}|| < 2 \times 10^{-11}$$

The quark propagators are computed with a Linux PC cluster (with 100 nodes), in which each node computes 1 column for 30 quark masses simultaneously.

Determination of a

We measure the pion time correlation function

$$\begin{aligned} C_\pi(t) &= \left\langle \sum_{\vec{x}} \mathrm{tr}\left\{ \gamma_5 (D_c + m_q)^{-1}(0,x) \gamma_5 (D_c + m_q)^{-1}(x,0) \right\} \right\rangle_U \\ &= \left\langle \sum_{\vec{x}} \mathrm{tr}\left\{ \left[(D_c + m_q)^{-1}{}^{ab}_{\alpha\beta}(x,0) \right]^* (D_c + m_q)^{-1}{}^{ab}_{\alpha\beta}(x,0) \right\} \right\rangle_U \end{aligned}$$

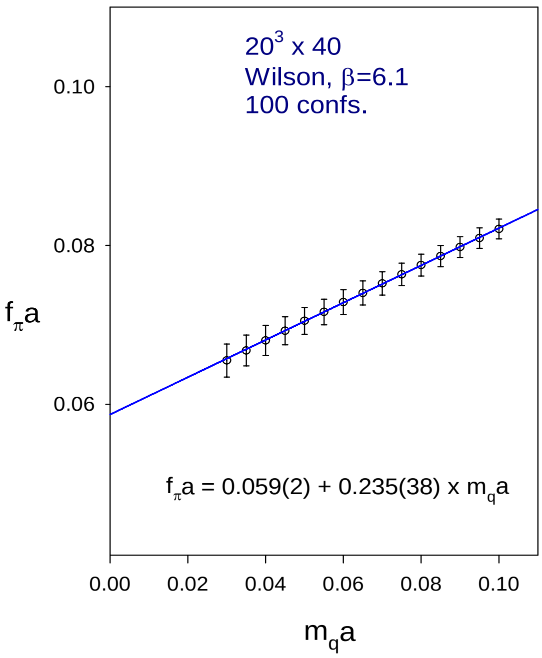
which is fitted to

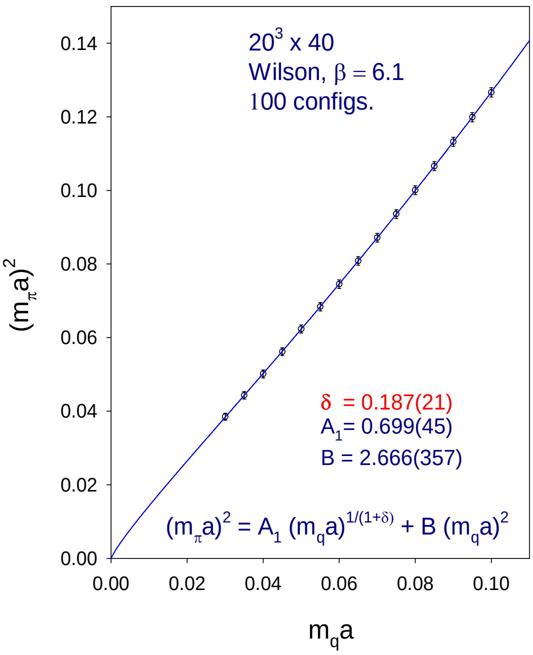
$$\frac{Z}{2m_\pi a}[e^{-m_\pi a t} + e^{-m_\pi a (T-t)}]$$

to extract pion mass $m_\pi a$ and decay constant

$$f_\pi a = 2 m_q a \frac{\sqrt{Z}}{(m_\pi a)^2} \ .$$

Determination of a (cont)

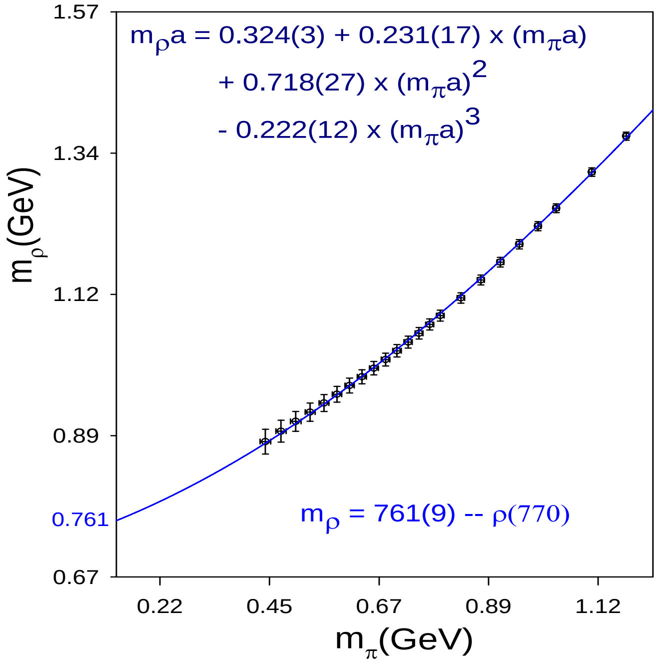




Taking $f_\pi a$ at $m_q a = 0$ equal to 0.131 GeV times a,

$$
\begin{aligned}
a^{-1} &= \frac{0.131}{f_0} = 2.237(76)\ (\text{GeV}) \\
a &= 0.088(3)\ \text{fm}
\end{aligned}
$$

Determination of m_s and m_c



Extract the mass of vector meson from

$$C_V(t) = \left\langle \frac{1}{3} \sum_{\mu=1}^{3} \sum_{\vec{x}} \mathrm{tr}\{\gamma_\mu (D_c + m_q)^{-1}_{x,0} \gamma_\mu (D_c + m_q)^{-1}_{0,x}\} \right\rangle_U$$

At $m_q a = 0.08$, $M_V = 1029(10)$ MeV, in good agreement with $\phi(1020)$. Thus we fix $m_s a = 0.08$. At $m_q a = 0.8$, $M_V = 3060(5)$ MeV, in good agreement with $J/\Psi(3097)$. Thus, we fix $m_c a = 0.8$.

Pseudoscalar decay constant

The decay constant f_P for a charged pseudoscalar meson P is defined by

$$\langle 0|A_\mu(0)|P(\vec{q})\rangle = f_P q_\mu$$

where $A_\mu = \bar{q}\gamma_\mu\gamma_5 Q$ is the axial-vector part of the charged weak current after a CKM matrix element $V_{qq'}$ has been removed.

Using $\partial_\mu A_\mu = (m_q + m_Q)\bar{q}\gamma_5 Q$, one obtains

$$f_P = (m_q + m_Q)\frac{|\langle 0|\bar{q}\gamma_5 Q|P(\vec{0})\rangle|}{m_P^2}$$

Pseudoscalar decay constant (cont)

We measure the pseudoscalar time-correlation function

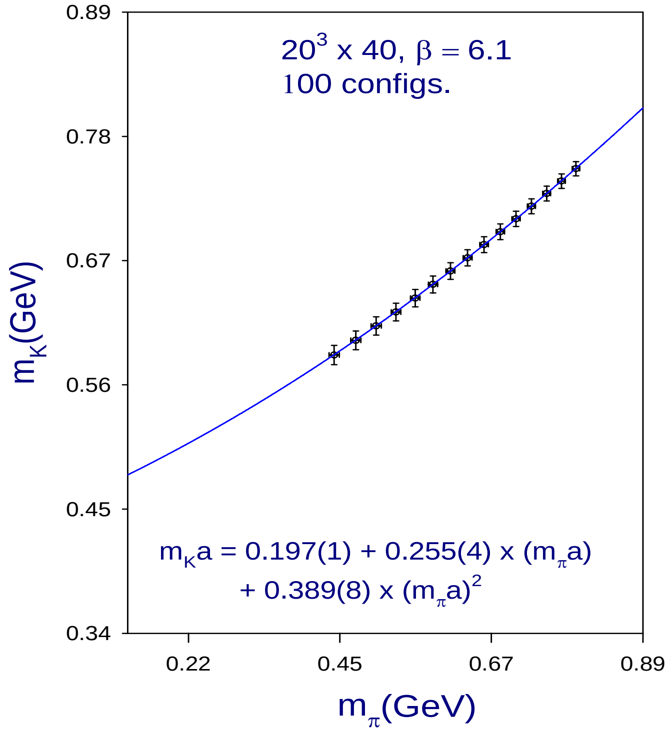
$$
\begin{aligned}
C_P(t) &= \left\langle \sum_{\vec{x}} \mathrm{tr}\Big\{ \gamma_5 (D_c + m_Q)^{-1}(0,x) \gamma_5 (D_c + m_q)^{-1}(x,0) \Big\} \right\rangle_U \\
&= \left\langle \sum_{\vec{x}} \mathrm{tr}\Big\{ \Big[(D_c + m_Q)^{-1}{}^{ab}_{\alpha\beta}(x,0) \Big]^* (D_c + m_q)^{-1}{}^{ab}_{\alpha\beta}(x,0) \Big\} \right\rangle_U
\end{aligned}
$$

which is fitted to

$$\frac{Z}{2m_P a}[e^{-m_P a t} + e^{-m_P a (T-t)}]$$

to extract pseudoscalar mass $m_P a$ and decay constant

$$f_P a = (m_q + m_Q) a \frac{\sqrt{Z}}{(m_P a)^2} \ .$$

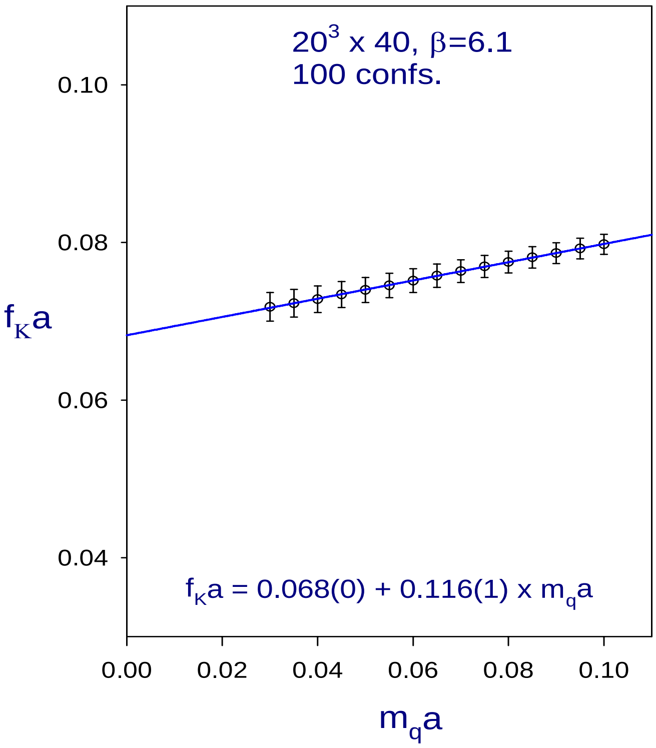


The data of $m_K a$ can be fitted by

$$m_K a = 0.197(1) + 0.255(4)(m_\pi a) + 0.389(8)(m_\pi a)^2 \ .$$

At $m_\pi = 135$ MeV, $m_K = 478(16)$ MeV. [PDG: $m_K = 495$ MeV]

f_{K+}

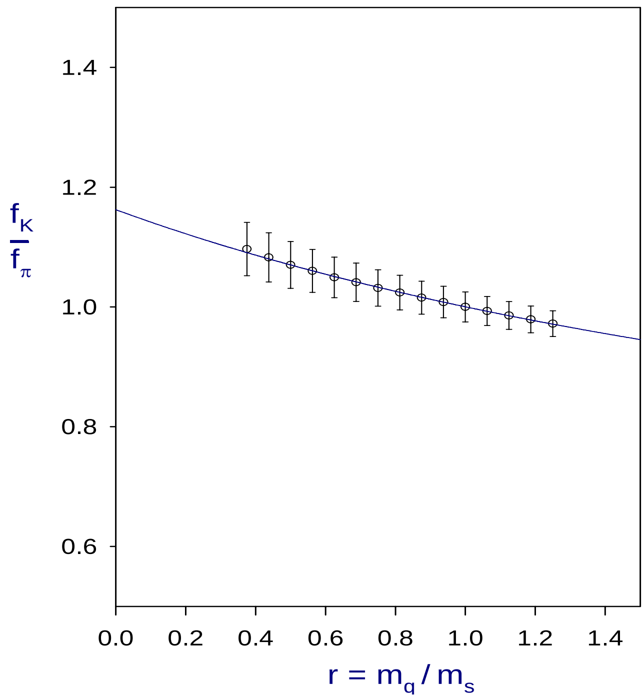


The data of $f_K a$ is well fitted by

$$f_K a = 0.068(0) + 0.116(1) \times (m_q a)$$

At $m_q a = 0$, $f_{K^+} = 150(6)$ MeV. [PDG: $f_{K^+} = 159.8(1.4)(0.44)$]

f_K/f_π



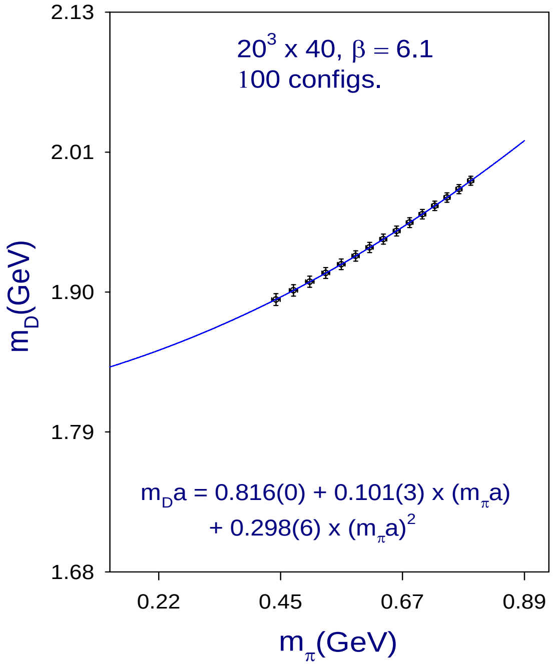
The data of f_{K^+}/f_π can be fitted by

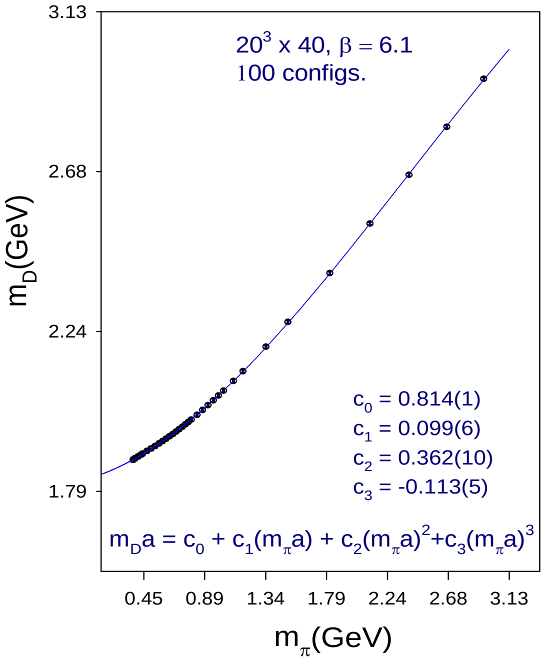
$$0.669(22) + \frac{0.931(116)}{1.813(163) + r}$$

At $r = 1/26$, it gives $f_{K^+}/f_\pi = 1.17(6)$ [$\sim$ 1.22 (PDG)]

$f_\pi = 131$ MeV $(input) \Rightarrow f_{K^+} = 153(8)$ MeV

$D(1865)$, $D_s(1968)$ and $\eta_c(2980)$





The data of $m_D a$ ($0.03 \le m_q a \le 0.08$) is well fitted by

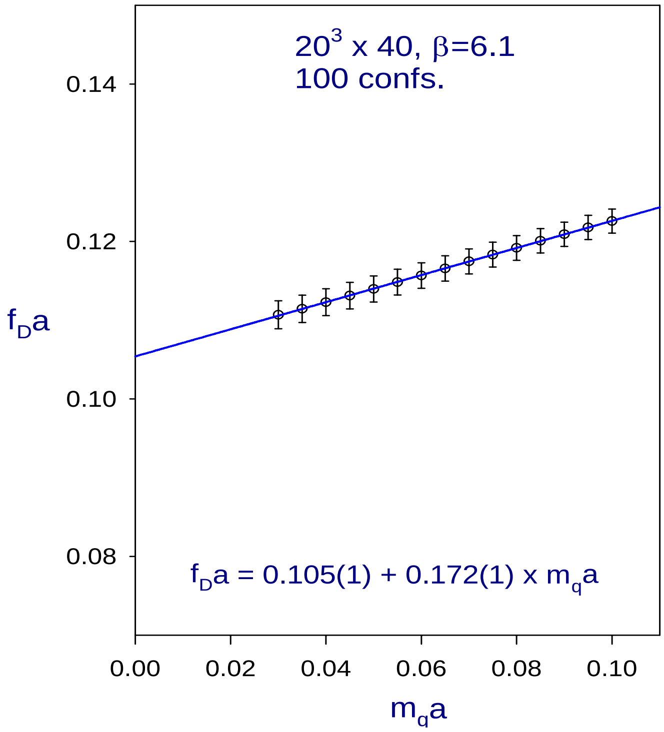
$$m_D a = 0.816(0) + 0.101(3) \times (m_\pi a) + 0.298(6) \times (m_\pi a)^2$$

At $m_\pi = 135$ MeV, it gives $m_D = 1842(15)$ MeV.

At $m_q a = m_s a = 0.08$, $m_{D_s} a = 0.8778(24) \Rightarrow m_{D_s} = 1964(5)$ MeV.

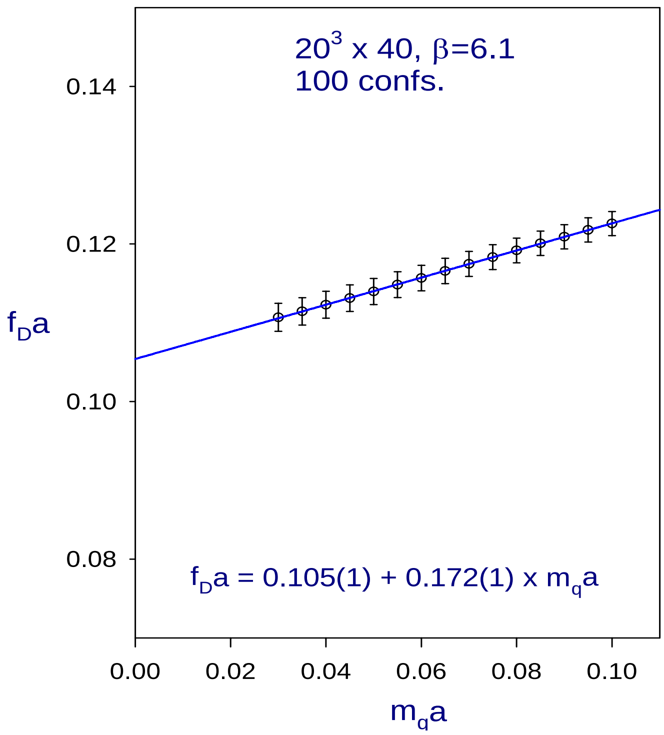
At $m_q a = m_c a = 0.80$, $m_{\eta_c} a = 1.3160(17) \Rightarrow m_{\eta_c} = 2944(4)$ MeV.

f_{D^+} and $f_{D_s^+}$



At $m_q a = m_s a = 0.08$, $f_{D_s} a = 0.119(2)$, it gives
$f_{D_s^+} = 266(10)$ MeV [PDG: $f_{D_s^+} = 267 \pm 33$ MeV]

f_{D^+} and $f_{D_s^+}$ (cont)



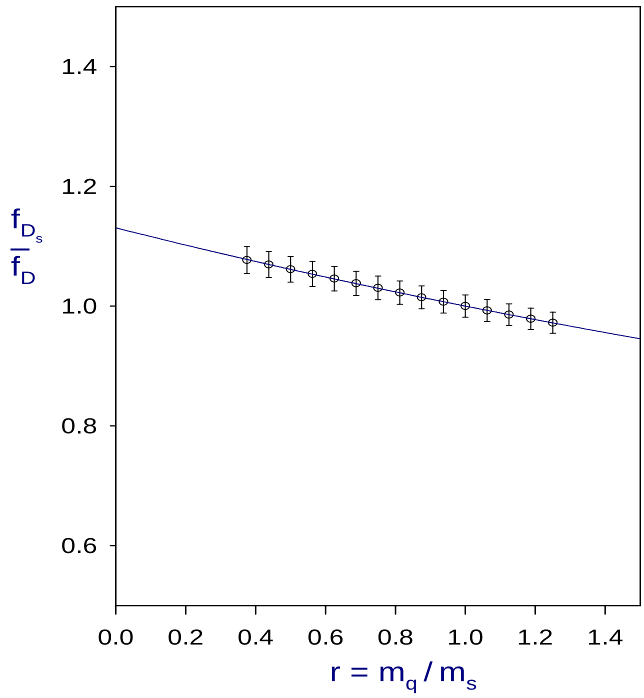
The data of $f_D a$ is well fitted by

$$f_D a = 0.105(1) + 0.172(1) \times (m_q a)$$

At $m_q a = 0$, $f_{D^+} = 235(8)$ MeV (hep-ph/0506266, June 26, 2005)

[CLEO: $f_{D^+} = 223 \pm 16^{+7}_{-9}$ MeV (Lepton-Photon, July 1, 2005)]

f_{D_s}/f_D



The data of $f_{D_s^+}/f_{D^+}$ can be fitted by

$$\frac{8.658(27)}{7.657(26) + r}$$

At $r = 1/26$, it gives $f_{D_s^+}/f_{D^+} = 1.13(2)$

Summary

Our predictive results (hep-ph/0506266) on June 26:

$$
\begin{aligned}
m_K &= 478 \pm 16 \pm 20 \text{ MeV} \\
m_{D_s} &= 1964 \pm 5 \pm 10 \text{ MeV} \\
m_D &= 1842 \pm 15 \pm 21 \text{ MeV} \\
f_{K^+} &= 152 \pm 6 \pm 10 \text{ MeV} \\
f_{D_s^+} &= 266 \pm 10 \pm 18 \text{ MeV} \\
f_{D^+} &= 235 \pm 8 \pm 14 \text{ MeV}
\end{aligned}
$$

where the first error is statistical, while the second is crude estimate of combined systematic uncertainty. Here the discretization error is estimated by comparing results to those from an ensemble of 221 gauge confs on $16^3 \times 32$ at $\beta = 6.0$ with $a^{-1} = 1.979(6)$ GeV.

[CLEO: $f_{D^+} = 223 \pm 16^{+7}_{-9}$ MeV, Lepton-Photon 2005, July 1]

[CLEO: $f_{D^+} = 222.6 \pm 16.7^{+2.8}_{-3.4}$ MeV, hep-ex/0508057]